**Title:**  Univariate and Multivariate Stochastic Orderings of Residual Lifetimes of Live Components in Sequential $(n-r+1)$-out-of-$n$Systems

**Authors:** Ghobad Barmalzan, Abedin Haidari and Narayanaswamy Balakrishnan

**Email:** ghobad.barmalzan@gmail.com

**Date:** January 2019

**Introduction**

In modeling technical systems, it is usually assumed that the failure of any component does not affect the performance of the remaining components. In many situations, however, a component failure will likely induce more stress on the remaining components thus affecting their reliability. As an example, consider a car with an eight-cylinder engine. After the failure of the first cylinder, the load on the remaining seven cylinders increases and consequently their lifetimes may become shorter. Another example comes from the nuclear power industry, wherein components (such as motor-operated valves for cooling) are usually redundantly added to safeguard against core meltdown. If the failure of one back-up system affects other systems, then the chance of a core meltdown can increase greatly; see Kvam and Pena (2005). For these reasons, a flexible model, which accommodates some form of dependence among the lifetimes of components, would be better employed when modeling the reliability of such systems. Sequential order statistics (SOS), initiated by Kamps (1995) as an extension of the usual order statistics, are useful in describing the ordered lifetimes of components in a system in which the failure of a component may affect the performance of the remaining components.

In many practical situations, when a technical system fails some of its components may still be alive and working. For example, in a series system with *n* components, after the first failure, *n* − 1 components of the system are still alive. It will naturally be of interest to study the distributional properties of the residual lifetimes of these live components. Note that if the original lifetimes of components follow an exponential population, due to its memoryless property, it is evident that the residual lifetimes of live (RLL) components of the system are distributed exactly the same as the original lifetimes. But, in general, when the components are not exponentially distributed, little is known about the RLL components. So, it will be of interest to establish some general distributional properties of the RLL components. Bairamov and Arnold (2008) studied distributional properties of the RLL units for the usual (*n*−*r*+1)-out-of- *n* system and obtained its joint distribution. They also characterized the exponential lifetime distribution under some conditions, and showed that under some well-known ageing properties of the original component lifetime distribution, one can compare the RLL components with the original lifetimes in terms of the usual stochastic ordering. Gurler (2012) extended the results of Bairamov and Arnold (2008) from the usual (*n* − *r* + 1)-out-of-*n* system to the sequential *(n*−*r*+1*)*-out-of-*n* system. More recently, Balakrishnan *et al.* (2014) established some ageing properties and stochastic orderings for the RLL components of the usual *(n* − *r* + 1*)*-out-of-*n* system with i.i.d. components and improved some of the results of Bairamov and Arnold (2008).

**Results**

In this research, we focus on the RLL components of a sequential (*n*−*r* +1)-out-of-*n* system. We establish some univariate stochastic ordering results. A characterization result for the exponential distribution based on uncorrelated RLL components and some ageing properties of the RLL components are discussed in Research. The usual multivariate stochastic order between the RLL components of two sequential *(n*−*r*+1*)*-out-of-*n* systems is established.